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A non-linear realisation of local internal supersymmetry

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Abstract. We propose a SU(2/1) local internal supersymmetry model with positive-definite kinetic terms for the gauge fields. The model is constructed by introducing additional scalar fields, according to the approach of the non-linear realisation of internal supersymmetry. The Lagrangian of the gauge fields is then given by using a scalar-field-dependent metric instead of the Killing metric. The system consisting of the gauge fields and scalar fields can be understood as a non-linear σ model coupled to gauge fields. The relation of our model to the massive Yang-Mills model is also discussed.

1. Introduction

Over the past ten years, local internal supersymmetry models (gauge theories of internal supersymmetry) based on the supergroup SU(2/1) have been investigated repeatedly by several authors in order to construct a unified electroweak theory with only one coupling constant [1-6].

The SU(2/1) model has the following interesting features. (a) The generators of SU(2/1), normalised by traces, fix the Weinberg angle to $\theta_W = 30°$, which is close to the present experimental value. (b) In addition to the triplet (fundamental) representation fit for the assignment of lepton states $(\nu_e^0, e_L^-/e_R^-)$, there exists the quartet representation of SU(2/1) which fits the assignment of quark states $(u_L^{2/3}, d_L^{-1/3}/u_R^{2/3}/d_R^{-1/3})$. (c) Goldstone-Higgs-like scalar fields and their interactions can be derived from SU(2/1) electroweak theories realised in a higher-dimensional spacetime. Extensions of the SU(2/1) electroweak theories to those including the strong interaction have been made by Taylor, Dondi and Jarvis and Ne'eman and Sternberg based on the supergroup SU(5/1) [7-9]. Some representations of SU(5/1) successfully reproduce the correct quantum numbers of leptons and quarks in different assignments from the conventional SU(5) model. Furthermore, to deal with the generations, an extended model based on SU(7/1) has also been investigated by Ne'eman and Thierry-Mieg [10].

In spite of the attractive properties mentioned above, the SU(2/1) model has the following difficulties [11]. First, since SU(2/1) is a Lie supergroup, the parameters for transformations caused by the odd generators, the generators of SU(2/1) other than those of subgroup SU(2) × U(1), are Grassmann numbers. This means that gauge fields associated with the odd generators are anticommuting vector fields with the wrong spin statistics. Correspondingly, the SU(2) doublet (ν_e^0, e_L^-) and the SU(2) singlet e_R^- assigned to the SU(2/1) triplet should have statistics opposite to each other, since anticommuting gauge fields propagate between (ν_e^0, e_L^-) and e_R^-. Hence, if (ν_e^0, e_L^-)

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are the left-handed components of the ordinary neutrino and the ordinary electron, then the singlet e_R^- must have the wrong spin statistics. Further, unlike the standard model, e_R^- has left chirality, since SU(2/1) commutes with the Lorentz group.

The second difficulty is that the Killing metric of SU(2/1) is not positive-definite for SU(2) × U(1), owing to the nature of the supertrace in the definition of the Killing metric. Therefore, the kinetic terms of SU(2) gauge fields and that of the U(1) gauge field have signs opposite to each other, and hence negative energies arise. Similar difficulties also arise in local internal supersymmetry models based on the supergroup SU(m/n) [12].

The same difficulty (the indefinite metric problem) is encountered when we deal with gauge theories of internal symmetry based on a non-compact Lie group such as SL(n, $\boldsymbol{C}$). In non-compact Lie groups, the Killing metric is also non-positive-definite; however, it is known that this difficulty can be avoided by replacing the Killing metric K_{AB} with a positive-definite metric $G_{AB}(\phi)$, which is defined with scalar fields introduced additionally [13, 14].

The purpose of this paper is to construct a local internal supersymmetry model in which the indefinite metric problem is avoided with the help of additional scalar fields.

In the next section, we discuss the non-linear realisation of a Lie supergroup, G, with a subgroup, H, and introduce the scalar fields ϕ^a as variables which parametrise the coset (super) space G/H [15]. There, we construct the Lagrangian for the fields ϕ^a invariant under global G transformations [16]. In § 3, we discuss the local G transformations and construct the Lagrangian of a system consisting of a gauge field and scalar fields. In § 4, we study the total Lagrangian in the unitary gauge and show that the Lagrangian of scalar fields is then reduced to the mass terms of gauge fields. In § 5, we apply this method to the SU(2/1) model and show that the positive-definite kinetic terms of gauge fields and the SU(2/1) symmetry are properly ordered. We also evaluate the Weinberg angle. Section 6 is devoted to a summary and discussion of our results. The appendix contains some geometrical formulae which are useful in this paper.

2. Non-linear realisation and global transformations

Any element g of a unitary supergroup G can be written as $\exp(\mathrm{i}X)$ by using an element X of the Lie superalgebra of G. In terms of the generators (the basis of the Lie superalgebra) $\{T_A\}$ ($A = 1, 2, \ldots, r$; $r = \dim \mathrm{G}$) of G, the X can be represented as $X = X^A T_A = (-1)^A T_A X^A$. Here, X^A are parameters (ordinary real numbers or real Grassmann numbers) and the exponent A in $(-1)^A$ is the 'Grassmann parity' defined as 0 (1) for ordinary (Grassmann) numbers. From the unitarity of G, the generators T_A satisfy the extended Hermiticity condition: $T_A{}^\dagger = (-1)^A T_A$ †. The structure of G is determined by the supercommutation relation

$$[T_A, T_B\}(\equiv T_A T_B - (-1)^{AB} T_B T_A) = \mathrm{i} f_{AB}{}^C T_C \tag{2.1}$$

where $f_{AB}{}^C$ are structure constants. The generators $\{T_A\}$ fall into two groups: the set $\{T_\alpha\}$ ($\alpha = 1, 2, \ldots, m$; $m = \dim \mathrm{H}$), which generates the subgroup H of G, and the

† The Hermitian conjugate († operation) and the complex conjugate (* operation) for the product of the elements of the Lie superalgebra are defined so that $(XY)^\dagger = Y^\dagger X^\dagger$, $(X^A T_A)^\dagger = T_A{}^\dagger X^{A*}$ and $(X^A Y^B)^* = Y^{B*} X^{A*}$.

remainder $\{T_a\}$ $(a = m+1, \ldots, r)$ associated with the coset superspace (homogeneous superspace) G/H [17, 18]. In this paper we restrict our discussion to the reductive coset superspace G/H, i.e. to the case of $f_{\alpha b}{}^{\gamma} = 0$.

Let us now consider the coset superspace G/H which is defined as the set of the (left) coset $gH = \{gh \mid \forall h \in H\}$ $(g \in G)$. From the definition, each of the cosets have no element in common, i.e. the cosets are disjoint†. Then we can introduce the coordinates $\{\phi^a\}$ on G/H which label the cosets in a one-to-one way. In this paper we treat the coordinates $\{\phi^a\}$ as $r-m$ scalar fields defined in the Minkowski spacetime. From each coset we choose a particular group element $v(\phi)$ $(\in G)$ called the 'coset representative'. Further, we assume that the coset representatives $v(\phi)$ constitute a smooth function $v = v(\phi)$ of $\{\phi^a\}$. All elements of the coset can be represented in the form $v(\phi)h$ $(h \in H)$. Since $v(\phi)$ is an element of G, the left product of $v(\phi)$ by an arbitrary element g $(\in G)$ lies in G, and hence the resultant element is in a coset specified by a coordinate $\{\phi'^a\}$:

$$gv(\phi(x)) = v(\phi'(x))h(\phi(x), g). \tag{2.2}$$

In general, h is a function of the scalar fields $\phi^a(x)$ and g. The explicit form of h is determined by (2.2), depending on the choice of elements $v(\phi)$. The transformation property $\phi^a(x) \to \phi'^a(x)$ of the scalar fields under the action of G is also determined by (2.2). The scalar fields $\phi^a(x)$ transform non-linearly under G, while, under the subgroup H, they transform linearly.

Considering the above properties of the coset superspace, we first construct the Lagrangian of the $\phi^a(x)$ which has the invariance under global G transformations. For this purpose, let us introduce the vector $e_\mu(\phi)$ $(\mu = 0, 1, 2, 3)$ defined by

$$e_\mu(\phi(x)) = (1/\mathrm{i}\kappa)v(\phi(x))^{-1}\partial_\mu v(\phi(x)) \tag{2.3}$$

where κ is a dimensionless constant and $\partial_\mu \equiv \partial/\partial x^\mu$. Since $v(\phi(x))^{-1}v(\phi(x+\delta x))$ $(\simeq 1 + \delta x^\mu v(\phi(x))^{-1}\partial_\mu v(\phi(x)))$ lies in G, $e_\mu(\phi)$ belongs to the Lie superalgebra of G. From (2.2), $e_\mu(\phi)$ transforms under global G transformations as

$$e_\mu(\phi) \to e_\mu(\phi') = he_\mu(\phi)h^{-1} + (1/\mathrm{i}\kappa)h\partial_\mu h^{-1}. \tag{2.4}$$

We note that, although g is independent of x, h is dependent on x through the $\phi^a(x)$; hence, there arises an inhomogeneous term in the transformation law. Thus, as in ordinary gauge theory, the invariant Lagrangian can be constructed by introducing the gauge field which cancels the inhomogeneous term in (2.4). We define the covariant derivative of $v(\phi)$ for the (local) H transformations by

$$\nabla_\mu v(\phi) = \partial_\mu v(\phi) - \mathrm{i}\kappa v(\phi)Q_\mu(x) \tag{2.5}$$

where Q_μ is the gauge field for H transformations, i.e. the element of the Lie superalgebra of H [16]. Hence, it can be expanded in terms of $\{T_\alpha\}$ as $Q_\mu(x) = Q_\mu^\alpha(x)T_\alpha$. We require that $\nabla_\mu v(\phi)$ has the same transformation property as $v(\phi)$ under the left product by g $(\in G)$, i.e.

$$g(\nabla_\mu v(\phi)) = (\nabla'_\mu v(\phi'))h(\phi, g) \tag{2.6}$$

from which the transformation of Q_μ must be

$$Q_\mu \to Q'_\mu = hQ_\mu h^{-1} + (1/\mathrm{i}\kappa)h\partial_\mu h^{-1}. \tag{2.7}$$

† The Lie supergroup G has the structure of the principal fibre bundle with the base space G/H, the typical fibre H and the projective map $\pi: G \to G/H$ defined by $\pi(g) = gH$. Furthermore, then, the coset representatives $v = v(\phi)$ define a 'section' on the bundle space G [13].

Therefore, if we define $p_\mu(\phi)$ by

$$p_\mu(\phi)=(1/\mathrm{i}\kappa)v(\phi)^{-1}\nabla_\mu v(\phi)=e_\mu(\phi)-Q_\mu \tag{2.8}$$

then it transforms homogeneously under the global G transformation as

$$p_\mu(\phi)\to p'_\mu(\phi')=hp_\mu(\phi)h^{-1}. \tag{2.9}$$

Now, let us introduce a Hermitian constant matrix η which is invariant under the action of the subgroup H, i.e.

$$h^{-1}\eta h=\eta \qquad \text{for all } h\in \mathrm{H} \tag{2.10a}$$

or equivalently

$$[\eta, T_\alpha]=0. \tag{2.10b}$$

Further, the $\{T_\alpha\}$ and $\{T_a\}$ are assumed to be orthogonal to each other in such a sense that the H-invariant metric I_{AB} defined by

$$I_{AB}=\tfrac{1}{2}(-1)^B[\mathrm{str}(T_A\eta T_B)+(-1)^{AB}\,\mathrm{str}(T_B\eta T_A)]$$
$$(=(-1)^{A+B+AB}I_{BA})) \tag{2.11}$$

satisfies $I_{\alpha a}=0$. The simplest example of η is the identity element in G; in this case, I_{AB} is reduced to the Killing metric: $K_{AB}\equiv(-1)^B\,\mathrm{str}(T_AT_B)$. For certain kinds of G and H, however, one can find a matrix η other than the identity element in G.

Using $p_\mu(\phi)$ and η, we can set up the Lagrangian of the $\phi^a(x)$ in the following form:

$$\mathscr{L}_{\phi,Q}=\tfrac{1}{2}\mu^2\,\mathrm{str}(p_\nu(\phi)\eta p^\nu(\phi))=\tfrac{1}{2}\mu^2 p_\nu{}^A(\phi)I_{AB}p^{\nu B}(\phi) \qquad p_\mu(\phi)=p_\mu{}^A(\phi)T_A \tag{2.12}$$

where μ is a constant with the dimension of mass. In consideration of (2.9) and (2.10a), the Lagrangian (2.12) is really invariant under the global G transformations. Now, we treat the gauge field Q_μ as an auxiliary field having no kinetic term; then, Q_μ can be expressed in terms of $\phi^a(x)$ by solving the constraint derived from the Lagrangian (2.12). This is done by decomposing $e_\mu(\phi)$ into the H part $e^-_\mu(\phi)$ and the G/H part $e^+_\mu(\phi)$ as (see (A10))

$$e_\mu(\phi)=e^-_\mu(\phi)+e^+_\mu(\phi) \tag{2.13}$$

where

$$e^-_\mu(\phi)\equiv e_\mu{}^\alpha(\phi)T_\alpha=(\partial_\mu\phi^b(x))e_b{}^\alpha(\phi)T_\alpha \tag{2.14a}$$

and

$$e^+_\mu(\phi)\equiv e_\mu{}^{(a)}(\phi)T_a=(\partial_\mu\phi^b(x))e_b{}^{(a)}(\phi)T_a. \tag{2.14b}$$

Substituting (2.13) into (2.12) and using the equation of motion $\partial\mathscr{L}_{\phi,Q}/\partial Q^\alpha_\mu=0$ with the orthogonal condition $I_{\alpha a}=0$, we have

$$Q_\mu=e^-_\mu(\phi) \tag{2.15}$$

from which (2.8) leads to

$$p_\mu(\phi)=e^+_\mu(\phi). \tag{2.16}$$

After eliminating Q_μ the Lagrangian (2.12) becomes

$$\mathscr{L}_\phi=\tfrac{1}{2}\mu^2\,\mathrm{str}(e^+_\nu(\phi)\eta e^{\nu+}(\phi))=\tfrac{1}{2}\mu^2 e_\nu{}^{(a)}(\phi)I_{ab}e^{\nu(b)}(\phi). \tag{2.17}$$

By using (2.14b), we can further rewrite (2.17) as

$$\mathscr{L}_\phi=\tfrac{1}{2}\mu^2\partial_\nu\phi^a(x)g_{ab}(\phi)\partial^\nu\phi^b(x) \tag{2.18}$$

where

$$g_{ab}(\phi) = (-1)^{b(b+d)} e_a{}^{(c)}(\phi) I_{cd} e_b{}^{(d)}(\phi)$$
$$(= (-1)^{a+b+ab} g_{ba}(\phi)). \tag{2.19}$$

The $\mathscr{L}_\phi$ in (2.18) is a familiar form of the Lagrangian of the 'non-linear σ model', in which the scalar fields parametrise some space (manifold) with a metric.

We finally check the equivalence of the transformation property on both sides of (2.15) and (2.16). From the reductivity condition $[T_\alpha, T_b\} = \mathrm{i} f_{\alpha b}{}^c T_c$, $hT_a h^{-1}$ can be expanded in the basis $\{T_a\}$. On the other side, $hT_\alpha h^{-1}$ and $h\partial_\mu h^{-1}$ are expanded in the basis $\{T_\alpha\}$. Therefore, substituting (2.13) into (2.4), we can decompose the transformation behaviour (2.4) into two pieces:

$$e_\mu^-(\phi) \to e_\mu^-(\phi') = h e_\mu^-(\phi) h^{-1} + (1/\mathrm{i}\kappa) h \partial_\mu h^{-1} \tag{2.20a}$$

$$e_\mu^+(\phi) \to e_\mu^+(\phi') = h e_\mu^+(\phi) h^{-1}. \tag{2.20b}$$

That is, e_μ^- and e_μ^+ transform identically as Q_μ and p_μ respectively.

3. Local transformations

3.1. *Lagrangian of scalar fields*

Next, let us consider the case where the transformations of the Lie supergroup G are spacetime dependent, i.e. the case of local G transformations. If an arbitrary element g of G has a spacetime dependence the transformation behaviour (2.4) of $e_\mu(\phi)$ will be modified so that

$$e_\mu(\phi) \to e_\mu(\phi') = h e_\mu(\phi) h^{-1} + (1/\mathrm{i}\kappa)[h\partial_\mu h^{-1} + h v(\phi)^{-1}(g^{-1}\partial_\mu g) v(\phi) h^{-1}]. \tag{3.1}$$

In the transformation behaviour (3.1), the third term is the inhomogeneous term arising from the spacetime dependence of G transformations. To construct an invariant Lagrangian we therefore need two kinds of gauge fields: one is the (auxiliary) gauge field Q_μ with the transformation behaviour (2.7) and the other is the gauge field A_μ of local G transformations, by which the third term of (3.1) is cancelled.

We now define the covariant derivative for G transformations:

$$D_\mu = \partial_\mu + \mathrm{i} q A_\mu(x) \qquad A_\mu(x) = A_\mu{}^B(x) T_B \tag{3.2}$$

where q is a coupling constant. Then, the covariant derivative of $v(\phi)$ can be obtained by replacing the ∂_μ with the D_μ in (2.5) as follows:

$$\mathscr{D}_\mu v(\phi) = D_\mu v(\phi) - \mathrm{i}\kappa v(\phi) Q_\mu(x)$$
$$(= (\nabla_\mu + \mathrm{i} q A_\mu(x)) v(\phi)). \tag{3.3}$$

If we assume that under G transformations the gauge field A_μ transforms as

$$A_\mu \to A'_\mu = g A_\mu g^{-1} + (1/\mathrm{i}q) g \partial_\mu g^{-1} \tag{3.4}$$

$\mathscr{D}_\mu v(\phi)$ has the same transformation property as $v(\phi)$ with respect to the left product by $g(x)$ ($\in$G). Thus, if we define

$$P_\mu(\phi) = (1/\mathrm{i}\kappa) v(\phi)^{-1} \mathscr{D}_\mu v(\phi) = E_\mu(\phi) - Q_\mu \tag{3.5}$$

with

$$E_\mu(\phi) \equiv (1/\mathrm{i}\kappa) v(\phi)^{-1} D_\mu v(\phi) = e_\mu(\phi) + (q/\kappa) v(\phi)^{-1} A_\mu v(\phi) \tag{3.6}$$

one can find that P_μ has the same transformation behaviour as p_μ, i.e.

$$P_\mu(\phi) \to P'_\mu(\phi') = hP_\mu(\phi)h^{-1}. \tag{3.7}$$

As in the case of global G transformations, we can set up the Lagrangian of $\phi^a(x)$, invariant under local G transformations, as

$$\mathcal{L}_{\phi,Q} = \tfrac{1}{2}\mu^2 \operatorname{str}(P_\nu(\phi)\eta P^\nu(\phi)) = \tfrac{1}{2}\mu^2 P_\nu{}^A(\phi) I_{AB} P^{\nu B}(\phi) \qquad P_\mu(\phi) = P_\mu{}^A(\phi) T_A. \tag{3.8}$$

Decomposing, here, E_μ into the H part E_μ^- and the G/H part E_μ^+, and using the equation of motion $\partial\mathcal{L}_{\phi,Q}/\partial Q_\mu{}^\alpha = 0$, we can easily get

$$Q_\mu = E_\mu^-(\phi) \equiv E_\mu{}^\alpha(\phi) T_\alpha = [(D_\mu\phi^b(x)) e_b{}^\alpha(\phi) + q\kappa^{-1} A_\mu{}^B \Omega_B{}^\alpha(\phi)] T_\alpha. \tag{3.9}$$

Then, (3.5) leads to

$$P_\mu(\phi) = E_\mu^+(\phi) \equiv E_\mu{}^{(a)}(\phi) T_a = (D_\mu\phi^b(x)) e_b{}^{(a)}(\phi) T_a \tag{3.10}$$

where the covariant derivative of the $\phi^a(x)$ is defined by

$$D_\mu\phi^a(x) = \partial_\mu\phi^a(x) + q\kappa^{-1} A_\mu{}^B(x) L_B{}^a(\phi). \tag{3.11}$$

Here we have used (A11) and (A12). Substituting (3.10) into (3.8), the Lagrangian (3.8) becomes

$$\mathcal{L}_\phi = \tfrac{1}{2}\mu^2 \operatorname{str}(E_\nu^+(\phi)\eta E^{\nu+}(\phi)) = \tfrac{1}{2}\mu^2 E_\nu{}^{(a)}(\phi) I_{ab} E^{\nu(b)}(\phi) \tag{3.12}$$

$$= \tfrac{1}{2}\mu^2 D_\nu\phi^a(x) g_{ab}(\phi) D^\nu\phi^b(x). \tag{3.13}$$

The $\mathcal{L}_\phi$ is nothing other than the Lagrangian of the non-linear σ model coupled with a gauge field. As in the case of global G transformations, we can verify that E_μ^- and E_μ^+ transform identically as Q_μ and P_μ, respectively.

3.2. *Lagrangian of the gauge field*

Let us construct a Lagrangian of the gauge field A_μ. From the covariant derivative (3.2), the field strength of the gauge field A_μ is defined by

$$F_{\mu\nu} = (1/\mathrm{i}q)[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu + \mathrm{i}q[A_\mu, A_\nu] \tag{3.14}$$

or, for the components,

$$F_{\mu\nu}{}^B = \partial_\mu A_\nu{}^B - \partial_\nu A_\mu{}^B + qA_\mu{}^C A_\nu{}^D f_{DC}{}^B \qquad F_{\mu\nu} = F_{\mu\nu}{}^B T_B. \tag{3.15}$$

From the transformation behaviour (3.4) we can show that

$$F_{\mu\nu} \to F'_{\mu\nu} = gF_{\mu\nu}g^{-1}. \tag{3.16}$$

Then the usual form of the Lagrangian, invariant under the transformation (3.16), is

$$-\tfrac{1}{4}\operatorname{str}(F_{\mu\nu}F^{\mu\nu}) = -\tfrac{1}{4}F_{\mu\nu}{}^A K_{AB} F^{\mu\nu B} \qquad K_{AB} \equiv (-1)^B \operatorname{str}(T_A T_B). \tag{3.17}$$

However, if there exists a matrix η having the property (2.10) besides the unit matrix, then another form of the Lagrangian of A_μ becomes possible. Indeed, using the matrix η and the coset representatives $v(\phi)$, we can define the following Hermitian matrix:

$$s(\phi) = v(\phi)\eta v(\phi)^{-1} \tag{3.18}$$

to which, in view of (2.10a), one can verify the transformation behaviour

$$s(\phi) \to s(\phi') = g s(\phi) g^{-1}. \tag{3.19}$$

Therefore, by using $s(\phi)$, we can set up another type of Lagrangian [13] as

$$\mathscr{L}_A = -\tfrac{1}{4}\,\mathrm{str}(F_{\mu\nu} s(\phi) F^{\mu\nu}) = -\tfrac{1}{4} F_{\mu\nu}{}^A G_{AB}(\phi) F^{\mu\nu B} \tag{3.20}$$

where

$$\begin{aligned} G_{AB}(\phi) &= \tfrac{1}{2}(-1)^B[\mathrm{str}(T_A s(\phi) T_B) + (-1)^{AB}\,\mathrm{str}(T_B s(\phi) T_A] \\ &(= (-1)^{A+B+AB} G_{BA}(\phi)). \end{aligned} \tag{3.21}$$

The metric $G_{AB}(\phi)$ transforms, under the G transformation, as

$$G_{AB}(\phi) \to G_{AB}(\phi') = (-1)^{B(B+D)} D_A{}^C(g^{-1}) G_{CD}(\phi) D_B{}^D(g^{-1}) \tag{3.22}$$

and is related to I_{AB} by

$$G_{AB}(\phi) = (-1)^{B(B+D)} D_A{}^C(v(\phi)^{-1}) I_{CD} D_B{}^D(v(\phi)^{-1}). \tag{3.23}$$

Here, $D_A{}^B(v^{-1})$ is the matrix corresponding to v^{-1} in the adjoint representation of G (see (A13)). The Lagrangian (3.20) is invariant under local G transformations. If we choose the unit matrix as η, then obviously $s(\phi) = 1$; the Lagrangian (3.20) is reduced to the Lagrangian in (3.17).

Therefore, the total Lagrangian invariant under the local G transformation should be

$$\mathscr{L}_{\mathrm{tot}} = \mathscr{L}_A + \mathscr{L}_\phi \tag{3.24}$$

where $\mathscr{L}_\phi$ is the Lagrangian defined in (3.12) or (3.13). We note that the H-invariant metric I_{AB} may be positive-definite, unlike the Killing metric K_{AB}, and so the kinetic term of the gauge field in the Lagrangian (3.24) may also be positive-definite.

4. Unitary gauge

We now discuss a specific expression of the Lagrangian obtained by a special G transformation: $g_0(x) \equiv v(\phi(x))^{-1}$. Let us suppose that $g_0(x)$ puts $\{\phi^a\}$ in the place of $\{\phi_0^a\}$ according to (2.2): $g_0(x)v(\phi) = v(\phi_0)h(\phi, g_0) = 1$. Since the unit element, 1, lies in H, $v(\phi_0)$ is a coset representative of the coset 1H, and so we can put $v(\phi_0) = h(\phi, g_0) = 1$ without loss of generality. Therefore, the scalar fields $\phi^a(x)$ are 'gauged away' out of the Lagrangian by the following $g_0(x)$ transformations:

$$A_\mu \to \tilde{A}_\mu \equiv v(\phi)^{-1} A_\mu v(\phi) + (1/\mathrm{i}q) v(\phi)^{-1}\partial_\mu v(\phi) \tag{4.1}$$

$$E_\mu(\phi) \to (q/\kappa)\tilde{A}_\mu \qquad e_\mu(\phi) \to 0 \tag{4.2}$$

$$F_{\mu\nu} \to \tilde{F}_{\mu\nu} \equiv v(\phi)^{-1} F_{\mu\nu} v(\phi) = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu + \mathrm{i}q[\tilde{A}_\mu, \tilde{A}_\nu] \tag{4.3}$$

$$s(\phi) \to \eta \tag{4.4}$$

and

$$G_{AB}(\phi) \to I_{AB}. \tag{4.5}$$

Then, the total Lagrangian $\mathscr{L}_{\mathrm{tot}}$ is reduced to the following form:

$$\begin{aligned} \tilde{\mathscr{L}}_{\mathrm{tot}} &= -\tfrac{1}{4}\,\mathrm{str}(\tilde{F}_{\mu\nu}\eta\tilde{F}^{\mu\nu}) + \tfrac{1}{2}m^2\,\mathrm{str}(\tilde{A}_\mu^+ \eta \tilde{A}^{\mu +}) \qquad && \tilde{A}_\mu^+ \equiv \tilde{A}_\mu{}^b T_b \\ &= -\tfrac{1}{4}\tilde{F}_{\mu\nu}{}^A I_{AB} \tilde{F}^{\mu\nu B} + \tfrac{1}{2}m^2 \tilde{A}_\mu{}^a I_{ab} \tilde{A}^{\mu b} && m \equiv (q/\kappa)\mu \end{aligned} \tag{4.6}$$

which should be compared with the Lagrangian of massive Yang-Mills fields in the 'unitary gauge'. We note that the Lagrangian $\mathscr{L}_{\phi}$ in (3.12) has been reduced to the mass terms for the gauge fields $\tilde{A}^{a}_{\mu}$ (G/H part), while the other gauge fields $\tilde{A}^{\alpha}_{\mu}$ (H part) still remain massless. The longitudinal components of the Proca-type massive fields $\tilde{A}^{a}_{\mu}$ are supplied by the scalar fields ϕ^{a}.

Now, in the Lagrangian $\tilde{\mathscr{L}}_{\text{tot}}$, the underlying symmetry of the supergroup G looks like breaking down to that of the subgroup H. However, since $\tilde{\mathscr{L}}_{\text{tot}}$ is a form of $\mathscr{L}_{\text{tot}}$ in a specific gauge, the symmetry of G in the Lagrangian (4.6) is not broken in truth. Indeed, from (2.2) and (3.4), one can find that $\tilde{A}_{\mu}$ and $\tilde{F}_{\mu\nu}$ transform under G transformations, respectively, as

$$\tilde{A}_{\mu} \to \tilde{A}'_{\mu} = h\tilde{A}_{\mu}h^{-1} + (1/\mathrm{i}q)h\partial_{\mu}h^{-1} \tag{4.7}$$

$$\tilde{F}_{\mu\nu} \to \tilde{F}'_{\mu\nu} = h\tilde{F}_{\mu\nu}h^{-1}. \tag{4.8}$$

In particular, the G/H part $\tilde{A}^{+}_{\mu}$ of $\tilde{A}_{\mu}$ transforms homogeneously as

$$\tilde{A}^{+}_{\mu} \to \tilde{A}'^{+}_{\mu} = h\tilde{A}^{+}_{\mu}h^{-1} \tag{4.9}$$

and so, by virtue of (2.10a), the full symmetry of the Lagrangian under G transformations still survives in $\tilde{\mathscr{L}}_{\text{tot}}$.

Finally, we stress the following difference between our model and the ordinary Higgs models: in our model (i) Higgs scalar fields disappear in the unitary gauge and (ii) there are no potential terms which generate spontaneous symmetry breaking.

5. The SU(2/1) model

Until now, the Lie supergroup G and its subgroup H have not been limited to specific supergroups. In this section we take the Lie supergroup SU(2/1) and the Lie group SU(2)×U(1) as G and H respectively. Then, the coset superspace G/H (=SU(2/1)/SU(2)×U(1)) turns out to be the pure fermionic space parametrised by Grassmann numbers. One can also verify that this coset space is reductive.

The generators of SU(2/1) consist of $\{T_{\alpha}\}$ ($\alpha = 1, 2, 3, 4$) generators of the subgroup SU(2)×U(1), and $\{T_{a}\}$ ($a = 5, 6, 7, 8$) generators mixing SU(2) doublet and SU(2) singlet. In the fundamental representation those Hermitian generators $\{t_{A}\}$ ($t_{\alpha} \sim T_{\alpha}$, $t_{a} \sim \mathrm{i}T_{a}$) are (disregarding the normalisation and writing σ_i as the Pauli matrix where $i = 1, 2, 3$) given by:

$$t_i = \left[\begin{array}{c|c} \sigma_i & \\ \hline & 0 \end{array}\right] \qquad t_4 = \left[\begin{array}{cc|c} -1 & & \\ & -1 & \\ \hline & & -2 \end{array}\right]$$

$$t_5 = \left[\begin{array}{cc|c} & & 1 \\ & & 0 \\ \hline 1 & 0 & \end{array}\right] \qquad t_6 = \left[\begin{array}{cc|c} & & -\mathrm{i} \\ & & 0 \\ \hline \mathrm{i} & 0 & \end{array}\right] \tag{5.1}$$

$$t_7 = \left[\begin{array}{cc|c} & & 0 \\ & & 1 \\ \hline 0 & 1 & \end{array}\right] \qquad t_8 = \left[\begin{array}{cc|c} & & 0 \\ & & -\mathrm{i} \\ \hline 0 & \mathrm{i} & \end{array}\right]$$

to which one can verify the supertraceless condition: $\mathrm{str}(t_A)=0$ †. The diagonal matrices t_3 and t_4 reproduce the respective quantum numbers of the weak isospin and the weak hypercharge for $(\nu_e^0, e_L^-/e_R^-)$. Since $\mathrm{str}(t_i^2)>0$ and $\mathrm{str}(t_4^2)<0$, the Killing metric of $\mathrm{SU}(2)\times\mathrm{U}(1)$ is not positive-definite: this is the difficulty mentioned in the introduction. In what follows, however, we shall show that the indefiniteness property of the Killing metric is not a serious problem in the system described by the Lagrangian (3.24).

In the present case, the matrix η that has the properties (2.10) can be written in the form

$$\eta=\left[\begin{array}{cc|c} 1 & & \\ & 1 & \\ \hline & & \lambda \end{array}\right] \tag{5.2}$$

where λ is a real constant. From (2.11), we can find that $I_{ii}\sim\mathrm{str}(t_i\eta t_i)>0$, and $I_{44}\sim\mathrm{str}(t_4\eta t_4)=2-4\lambda$. Therefore, the $\mathrm{SU}(2)\times\mathrm{U}(1)$ part of the metric I_{AB} has definite sign provided $\lambda<\frac{1}{2}$. Taking the condition on λ and the property $T_A{}^\dagger=(-1)^A T_A$ into account, we here define the normalised generators $\{T_A\}$ as

$$\begin{aligned} &T_i=\frac{1}{\sqrt{2}}t_i \qquad T_4=\frac{1}{[2(1-2\lambda)]^{1/2}}t_4 \qquad \text{for } \lambda<\tfrac{1}{2} \\ &T_a=\frac{\mathrm{i}^r}{(1+\lambda)^{1/2}}t_a \qquad r=\begin{cases}1 & \text{for } -1<\lambda<\frac{1}{2}\\ 0 & \text{for } \lambda<-1.\end{cases}\end{aligned} \tag{5.3}$$

Then, I_{AB} has the following form:

$$I_{AB}=\left[\begin{array}{cccc|cc} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ \hline & & & & (-1)^r\sigma_2 & \\ & & & & & (-1)^r\sigma_2 \end{array}\right]. \tag{5.4}$$

In this case, the Lagrangian $\tilde{\mathscr{L}}_{\mathrm{tot}}$ in (4.6) becomes

$$\begin{aligned}\tilde{\mathscr{L}}_{\mathrm{tot}}=&-\tfrac{1}{4}(\tilde{F}_{\mu\nu}{}^i\tilde{F}^{\mu\nu i}+\tilde{F}_{\mu\nu}{}^4\tilde{F}^{\mu\nu 4})+\mathrm{i}(-1)^r(\tfrac{1}{2}\tilde{F}_{\mu\nu}{}^5\tilde{F}^{\mu\nu 6}-m^2\tilde{A}_\mu{}^5\tilde{A}^{\mu 6})\\ &+\mathrm{i}(-1)^r(\tfrac{1}{2}\tilde{F}_{\mu\nu}{}^7\tilde{F}^{\mu\nu 8}-m^2\tilde{A}_\mu{}^7\tilde{A}^{\mu 8}).\end{aligned} \tag{5.5}$$

Therefore, the kinetic terms for SU(2) and U(1) gauge fields have the same sign, and the anticommuting gauge fields become massive. The Lagrangian with positive-definite kinetic terms in an arbitrary gauge can also be obtained by using (3.23).

We next evaluate the Weinberg angle θ_W from the form of interaction between leptons and gauge fields. As mentioned in the introduction, the ordinary left-handed doublet (ν_e^0, e_L^-) and the ordinary right-handed singlet e_R^- cannot be assigned to the triplet representation of SU(2/1). Thus, in this paper, we regard (ν_e^0, e_L^-) and e_R^- as physical components of a pair of SU(2/1) triplets: $(\nu_e^0, e_L^-/\langle e_R^-\rangle_L^G)$ and $(\langle\nu_e^0\rangle_R^G, \langle e_L^-\rangle_R^G/e_R^-)$ [2, 3, 5]. Here, $\langle e_R^-\rangle_L^G$ and $(\langle\nu_e^0\rangle_R^G, \langle e_L^-\rangle_R^G)$ are left-handed and right-handed bosonic spinor fields with wrong spin statistics respectively.

† The supertrace is defined by $\mathrm{str}(\left[\begin{smallmatrix}A & \\ & B\end{smallmatrix}\right])=\mathrm{tr}\,A-\mathrm{tr}\,B$.

Taking account of the gauge invariance, the Lagrangian for the SU(2/1) triplets is given by

$$\mathscr{L}_\Psi = \mathrm{i} \sum_{k=\mathrm{L,R}} \bar{\Psi}_k \gamma^\mu D_\mu \Psi_k \tag{5.6}$$

where

$$\Psi_\mathrm{L} = \left[\begin{array}{c} \psi_\mathrm{L} \\ \hline \psi_\mathrm{L}^G \end{array}\right] = \left[\begin{array}{c} \nu_e^0 \\ e_\mathrm{L}^- \\ \hline \langle e_\mathrm{R}^- \rangle_\mathrm{L}^G \end{array}\right] \qquad \Psi_\mathrm{R} = \left[\begin{array}{c} \psi_\mathrm{R}^G \\ \hline \psi_\mathrm{R} \end{array}\right] = \left[\begin{array}{c} \langle \nu_e^0 \rangle_\mathrm{R} \\ \langle e_\mathrm{L}^- \rangle_\mathrm{R} \\ \hline e_\mathrm{R}^- \end{array}\right]$$

and D_μ is the covariant derivative defined in (3.2). The interaction terms of Ψ_k with A_μ^3 and A_μ^4 are

$$\begin{aligned} &\sum_{k=\mathrm{L,R}} q\bar{\Psi}_k(\gamma^\mu A_\mu{}^3 T_3 + \gamma^\mu A_\mu{}^4 T_4)\Psi_k \\ &\quad = \frac{q}{\sqrt{2}}\left(\bar{\psi}_\mathrm{L}\gamma^\mu A_\mu{}^3 \sigma_3 \psi_\mathrm{L} - \frac{1}{(1-2\lambda)^{1/2}}(\bar{\psi}_\mathrm{L}\gamma^\mu A_\mu{}^4 \psi_\mathrm{L} + 2\bar{\psi}_\mathrm{R}\gamma^\mu A_\mu{}^4 \psi_\mathrm{R})\right) \\ &\qquad + (\text{ghost terms}). \end{aligned} \tag{5.7}$$

Comparing (5.7) with those in the Weinberg-Salam model

$$\tfrac{1}{2}g[\bar{\psi}_\mathrm{L}\gamma^\mu A_\mu{}^3\sigma_3\psi_\mathrm{L} - (g'/g)(\bar{\psi}_\mathrm{L}\gamma^\mu B_\mu \psi_\mathrm{L} + 2\bar{\psi}_\mathrm{R}\gamma^\mu B_\mu \psi_\mathrm{R})] \qquad g = \sqrt{2}q \tag{5.8}$$

we have the Weinberg angle

$$\tan\theta_\mathrm{W} \equiv \frac{g'}{g} = \frac{1}{(1-2\lambda)^{1/2}} \tag{5.9}$$

where g and g' are the coupling constants of SU(2) and U(1) respectively.

We note here that the same Weinberg angle θ_W as (5.9) can also be obtained in the quartet representation, in which the quantum numbers of the weak isospin and weak hypercharge for the quark states $(u_\mathrm{L}^{2/3}, d_\mathrm{L}^{-1/3}/u_\mathrm{R}^{2/3}/d_\mathrm{R}^{-1/3})$ can be reproduced. Indeed, the generators T_3 and T_4 in that representation are, respectively, given by [2, 8, 9]

$$T_3^{\text{quartet}} = \frac{1}{\sqrt{2}}\left[\begin{array}{c|cc} \sigma_3 & & \\ \hline & 0 & \\ & & 0 \end{array}\right]$$

and (5.10)

$$T_4^{\text{quartet}} = \frac{1}{[2(1-2\lambda)]^{1/2}}\left[\begin{array}{cc|cc} -2b+1 & & & \\ & -2b+1 & & \\ \hline & & -2b+2 & \\ & & & -2b \end{array}\right]$$

where b is a real constant, and so the generators (5.10) with $b=\frac{1}{3}$ give the weak isospin and weak hypercharge for such quark states correctly. If we assign the quark states to the quartet representation of SU(2/1) in practice, it is necessary to prepare a pair of SU(2/1) quartets with statistics opposite to each other, as in the case of leptons. Then, from the interaction terms of quarks with gauge fields, the Weinberg angle is angle is determined again as (5.9) [2]. It should also be noticed that the quartet representation (5.10) with the value $b=\frac{2}{3}$, $b=1$ and $b=0$ reproduce the weak isospin and weak hypercharge for the antiquark states $(\bar{d}_{\mathrm{R}}^{1/3}, \bar{u}_{\mathrm{R}}^{-2/3}/\bar{d}_{\mathrm{L}}^{1/3}/\bar{u}_{\mathrm{L}}^{-2/3})$, the lepton states $(\nu_{e\mathrm{L}}^{0}, e_{\mathrm{L}}^{-}/\nu_{e\mathrm{R}}^{0}/e_{\mathrm{R}}^{-})$ and the antilepton states $(e_{\mathrm{R}}^{+}, \bar{\nu}_{e\mathrm{R}}^{0}/e_{\mathrm{L}}^{+}/\bar{\nu}_{e\mathrm{L}}^{0})$ respectively [9]. In each choice of b we can obviously obtain the same value of the Weinberg angle θ_{W}.

In our model the Weinberg angle is not predictable, owing to the undetermined real constant λ $(<\frac{1}{2})$. Further, the case $\lambda=-1$, corresponding to $\theta_{\mathrm{W}}=30°$, the theoretical value obtained in the original papers [1, 2], is not acceptable in our model, as can be seen from the normalisation (5.3).

6. Summary and discussion

We have constructed a new type of Lagrangian $\mathscr{L}_A$ for the gauge field A_μ of a local internal supersymmetry with the help of the metric $G_{AB}(\phi)$, which is a function of scalar fields $\phi^a(x)$ and plays a similar role of the Killing metric K_{AB}. The total Lagrangian $\mathscr{L}_{\mathrm{tot}}$ is obtained by adding the Lagrangian $\mathscr{L}_\phi$ of the scalar fields to $\mathscr{L}_A$.

The Lagrangian $\mathscr{L}_{\mathrm{tot}}$ is essentially the same as that of the non-linear σ model coupled to the gauge field. Especially for $H=\{1\}$ and $\eta=1$, the Lagrangian $\mathscr{L}_{\mathrm{tot}}$ is reduced to the Lagrangian of the massive Yang–Mills model, which was investigated in [19]. Further, in a specific guage (the unitary gauge), the G/H part of the gauge field A_μ becomes the Proca field by introducing the freedom of the scalar fields ϕ^a, while the H part of A_μ still remains massless in this gauge.

Using the method of § 3, we have succeeded in getting a SU(2/1)-invariant Lagrangian with positive-definite kinetic terms for the gauge fields. In our choice of the subgroup $H=\mathrm{SU}(2)\times\mathrm{U}(1)$, the anticommuting gauge fields (ghost fields) can acquire a mass, m. This means that, for a sufficiently large m, the full local symmetry described by SU(2/1) may be hidden in the low-energy region [20] in spite of the presence of the symmetry in the formalism†.

The Weinberg angle has been evaluated from the interaction terms of gauge fields with leptons and quarks. Then, in our model, the Weinberg angle is obtained, depending on a constant λ $(<\frac{1}{2})$ in the form $\tan\theta_{\mathrm{W}}=1/(1-2\lambda)^{1/2}$; although SU(2/1) is a simple supergroup, the Weinberg angle is not determined uniquely. In our model, we expect that λ (i.e. θ_{W}) will be fixed by other reasons.

In addition, in our formalism, there still remains the problem of the ghosts, which spoil the spin-statistics relation. We leave this problem for future work.

As for the case of the local internal supersymmetry model based on $\mathrm{SU}(m/n)$ $(m\geqslant n)$ [21] we point out the following: if we choose the maximum bosonic subgroup $\mathrm{SU}(m)\times\mathrm{SU}(n)\times\mathrm{U}(1)$ ($\mathrm{SU}(m)\times\mathrm{U}(1)$ if $m\geqslant 2$, $n=1$ and U(1) if $m=n=1$) as H, we can take the following diagonal matrix for η with the properties (2.10):

$$\eta=\left[\begin{array}{c|c} I_m & \\ \hline & \lambda I_n \end{array}\right] \tag{6.1}$$

where I_m (I_n) is a $m\times m$ $(n\times n)$ unit matrix, and λ is a real constant. Then, the Lagrangian with positive-definite kinetic terms for the gauge fields can be derived on

† If we choose the Lie group U(1), instead of $\mathrm{SU}(2)\times\mathrm{U}(1)$, as the subgroup H, the three ordinary gauge fields become massive in addition to the four anticommuting gauge fields. Thus, we may give mass to the W and Z bosons within the framework of this paper.

the condition that $\lambda < 1/m$ (for $n=1$), $\lambda < 0$ (for $n \geqslant 2$). In this choice of subgroup, the anticommuting gauge fields become massive.

We finally comment on the high-energy behaviour of the propagator of the gauge fields. The Feynman propagator for the massive gauge fields $A_\mu{}^a$ (the G/H part) can be derived from the quadratic part (the free part) of the Lagrangian (3.24) with the gauge fixing terms in the following form [19]:

$$-\frac{\mathrm{i}}{(2\pi)^4} I^{ab}\left[\frac{1}{k^2-m^2}\left(\eta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+\alpha\frac{k_\mu k_\nu}{(k^2)^2}\right] \tag{6.2}$$

where I^{ab} is the inverse of I_{ab} and α is a gauge parameter. It should be noticed that, unlike the propagator of the Proca field, the propagator of $A_\mu{}^a$ has the same high-energy behaviour as that of the massless gauge fields belonging to the H part. Therefore, in the SU(m/n) model, we may expect that the massive anticommuting gauge fields play the role of the 'regulators' for the ordinary massless gauge fields.

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Appendix

We here discuss a number of formulae needed for an understanding of the coset superspace G/H parametrised by $\{\phi^a\}$ $(\phi^a\phi^b=(-1)^{ab}\phi^b\phi^a)$ [17, 18]. As seen from (2.2) the coset representatives $v(\phi)$ transform under the action of G according to

$$v(\phi)\to v(\phi')=gv(\phi)h(\phi, g)^{-1}. \tag{A1}$$

Now, let us consider the infinitesimal G transformations

$$g=1-\mathrm{i}q\varepsilon^A T_A \tag{A2}$$

where the ε^A are infinitesimal parameters which depend on (x^μ) for the local G transformation. Under the transformations (A2), the scalar fields $\phi^a(x)$ transform as

$$\phi^a\to\phi'^a=\phi^a-q\kappa^{-1}\varepsilon^A L_A{}^a(\phi). \tag{A3}$$

Correspondingly, $h(\phi, g)$ is represented in the following form:

$$h(\phi, g)=1-\mathrm{i}q\varepsilon^A\Omega_A{}^\alpha(\phi)T_\alpha. \tag{A4}$$

The vectors $L_A{}^a(\phi)$ are Killing vectors. $\Omega_A{}^\alpha(\phi)$ are called H compensators. Substituting (A2)-(A4) into (A1), one can find that

$$\begin{aligned}\delta v(\phi)&=v(\phi')-v(\phi)\\&=-q\kappa^{-1}\varepsilon^A L_A v(\phi)=\mathrm{i}q\varepsilon^A(-T_A v(\phi)+v(\phi)\Omega_A{}^\alpha(\phi)T_\alpha)\end{aligned} \tag{A5}$$

where $L_A \equiv L_A{}^a(\phi)\partial_a$, and $\partial_a \equiv \vec{\partial}/\partial\phi^a$ is the left derivative with respect to ϕ^a. From $(g_1g_2)v(\phi) = g_1(g_2v(\phi))$, we obtain the following relations:

$$L_A\Omega_B{}^\gamma - (-1)^{AB}L_B\Omega_A{}^\gamma = \kappa[f_{AB}{}^C\Omega_C{}^\gamma + (-1)^{\alpha B}\Omega_A{}^\alpha\Omega_B{}^\beta f_{\beta\alpha}{}^\gamma] \tag{A6}$$

$$[L_A, L_B\} = \kappa f_{AB}{}^C L_C \tag{A7}$$

where we have used (A5) and the supercommutation relation

$$[T_A, T_B\} = \mathrm{i} f_{AB}{}^C T_C. \tag{A8}$$

In terms of the left derivative, the 1-form on G/H, which is related to $e_\mu(\phi)$ in (2.3) by $e(\phi) = dx^\mu e_\mu(\phi)$, is defined by

$$e(\phi) = e^A(\phi)T_A = (1/\mathrm{i}\kappa)v(\phi)^{-1}dv(\phi) \qquad d \equiv d\phi^a\partial_a = dx^\mu\partial_\mu. \tag{A9}$$

For the components, this equation is expressed as

$$e_b(\phi) = e_b{}^\alpha(\phi)T_\alpha + e_b{}^{(a)}(\phi)T_a = (1/\mathrm{i}\kappa)v(\phi)^{-1}\partial_b v(\phi). \tag{A10}$$

The $e_b{}^{(a)}(\phi)$ and $e_b{}^\alpha(\phi)$ are called vielbeins and H connections respectively†. By multiplying $L_A{}^a(\phi)$ with (A10) and using (A5), we obtain the following relations:

$$L_A{}^a(\phi)e_a{}^\beta(\phi) = D_A{}^\beta(v(\phi)^{-1}) - \Omega_A{}^\beta(\phi) \tag{A11}$$

$$L_A{}^a(\phi)e_a{}^{(b)}(\phi) = D_A{}^b(v(\phi)^{-1}) \tag{A12}$$

where the matrix $D_A{}^B$ is the adjoint representation of G defined by

$$gT_Ag^{-1} = D_A{}^B(g)T_B \qquad g \in \mathrm{G}. \tag{A13}$$

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† When we need to distinguish the indices of the local frame basis from the indices of the coordinate basis, we shall write the former with parentheses.

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